Origin of resonances in the chiral unitary approach

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We study the origin of the resonances associated with pole singularities of the scattering amplitude in the chiral unitary approach. We propose a "natural renormalization" scheme using the low-energy interaction and the general principle of the scattering theory. We develop a method to distinguish dynamically generated resonances from genuine quark states [Castillejo-Dalitz-Dyson (CDD) poles] using the natural renormalization scheme and phenomenological fitting. Analyzing physical meson-baryon scatterings, we find that the $\Lambda(1405)$ resonance is largely dominated by the meson-baryon molecule component. In contrast, the $N(1535)$ resonance requires a sizable CDD pole contribution, while the effect of the meson-baryon dynamics is also important.

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I. INTRODUCTION

Chiral symmetry is one of the guiding principles for studying hadron physics based on the underlying theory of QCD. The chiral perturbation theory [1, 2, 3, 4, 5] enables us to study low-energy hadron dynamics systematically. By construction, however, perturbative calculations cannot be applied to the system with bound states and/or resonances. For instance, the leading order term of the chiral perturbation theory describes well the πN scattering lengths [6, 7], while it cannot reproduce either the πN scattering amplitude around the Δ resonance energy or the $\bar{K}N$ scattering length due to the presence of the $\Lambda(1405)$ resonance below the threshold. To describe the latter system in the chiral effective theory, the resonances can be either introduced as elementary fields in the Lagrangian or generated dynamically in hadron scattering. In general, they can also mix. The clarification of these dynamics is one of issues that we discuss in this paper.

Recent developments in the study of resonance scattering based on chiral dynamics have been made; the implementation of the unitarity condition on the scattering amplitude leads to the nonperturbative resummation of the s-channel diagrams, generating the resonance pole in the amplitude dynamically. This chiral unitary approach was successfully applied to the scattering of the pseudoscalar meson with octet baryons [8, 9, 10, 11, 12, 13], with pseudoscalar mesons [14, 15, 16, 17], with decuplet baryons [18, 19], with vector mesons [20, 21], and with heavy flavored hadrons [22, 23, 24], thanks to the

dominant contribution from the model-independent low-energy interaction [25, 26]. These studies reproduce many scattering observables as well as the properties of the observed resonances.

Despite the remarkable success of the chiral unitary approach, the origin of the resonances is not well understood, especially for the baryonic sector. One simply expects that the resonances found in this approach are quasibound states of a meson and a baryon generated by their two-body interaction. Hereafter we call this by the meson-baryon picture of the resonance. This picture may be in contrast to the description of resonances as genuine quark states. Such a state is generally called the Castillejo-Dalitz-Dyson (CDD) pole [27, 28], which is not generated in the dynamics of the meson-baryon scattering, but has some different origins.[1] The importance of the CDD pole in the chiral unitary approach was first pointed out and discussed in Ref. [29].

In most cases, the CDD pole is introduced explicitly as an elementary field in the chiral perturbation theory [30, 31] or in the unitarized framework [29, 32, 33, 34]. There are, however, some cases in which the CDD pole contribution is hidden in the model parameters. For instance, in $\pi\pi$ scattering, it has been argued that the pole for the ρ meson is attributed to contact terms in the higher order Lagrangian [17], which is known to be-

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[1] Strictly speaking, a pole singularity of scattering amplitude for an elementary particle is different from the pole originally introduced in Ref. [27], which gives a pole of the inverse amplitude. The presence of the original CDD pole was later interpreted as an independent particle participating in the scattering; see, e.g., Ref. [28]. We will nevertheless use the term "CDD pole" to indicate the pole of the elementary particle for simplicity.

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have as a contracted resonance propagator in the chiral perturbation theory [5, 35]. Hence, the nature of the ρ meson is considered to be of the CDD pole, presumably originated from the quark dynamics. This observation is in accordance with the study of the large N_c limit and the N_c scaling, where the pole of the ρ meson behaves as a $\bar{q}q$ resonance rather than a two-meson quasibound state [29, 36].

Furthermore, it is also possible to have both CDD pole and dynamical state in one system [29, 33]. In this case, the two components will be mixed in physical states. An example of the mixed situation has been studied in Ref. [37]. There they studied the coupling property of the introduced field, which turned out to be similar to the corresponding physical resonance in full amplitude. In general, such a comparison of the couplings is useful in studying the origin of the resonance.

In this paper, we study the origin of the resonances in chiral dynamics, paying attention to the renormalization procedure. In the chiral unitary approach, we need to introduce renormalization parameters (subtraction constants) in order to tame the divergence in loop integrals, which have been used to fit experimental data [9, 38, 39, 40]. Here we propose a different strategy: determining the subtraction constant first to study the structure of the resonances. Namely, we investigate whether the baryonic resonances obtained in the chiral unitary approach are purely dynamically generated resonances by meson-baryon scatterings or they have some components other than the dynamical one. For this purpose, we develop a renormalization scheme based on purely a theoretical argument to exclude the CDD pole contribution in the loop function. We introduce the following two requirements: (1) the scattering equation shares a common feature with ordinary quantum mechanical problems based on the Schrödinger equation, and (2) the obtained scattering amplitude is consistent with the low-energy interaction at a certain kinematic point. With these conditions, we determine the value of the subtraction constant uniquely for the single-channel scattering system without CDD poles. We call this scheme "natural renormalization," which specifies a standard value of the subtraction constant. Having this scheme, we will discuss the meaning of the subtraction constant, which is different from the standard value, in what follows.

Next we consider the scattering amplitude in comparison with experimental data, and propose a method to extract the low-energy structure of the amplitude in the natural renormalization scheme. From the viewpoint of renormalization, we first note that the change of the subtraction constant can be absorbed into the change of the interaction kernel, once the experimental input is given. If the resonance is dominated by the meson-baryon component, experimental data are well reproduced in the natural renormalization scheme with the interaction kernel without the CDD pole contribution. If the experimental amplitude requires a large contribution from the CDD

pole, one has to introduce its effect either in the subtraction constant or in the kernel interaction. In one way, we can reproduce experimental data by suitably choosing the subtraction constant, but keeping the interaction kernel unchanged. We find that this phenomenological amplitude can be equivalently expressed by the natural value of the subtraction constant and the interaction with explicit contribution of the CDD pole. In this way, the origin of the resonances can be studied, making use of the natural renormalization scheme and the experimental input.

This paper is organized as follows. In Sec. II, we describe the formulation of the chiral unitary approach for a single channel scattering, based on the N/D method. In Sec. III, we discuss the properties of the loop function theoretically in the meson-baryon picture. We derive the natural value for the subtraction constant from the consistency with the general principle and low-energy interaction. In Sec. IV, we present an interpretation of phenomenological fitting to experimental data. From the viewpoint of the renormalization, we analyze the deviation of the subtraction constant from the natural value. We then generalize the framework to the coupled-channel scattering problem in Sec. V and perform numerical analysis in Sec. VI for the strangeness $S = -1$ and $S = 0$ meson-baryon scatterings. The obtained results are discussed in connection with related works in Sec. VII, and concluding remarks are given in the last section.

II. CHIRAL UNITARY APPROACH

A. Unitarity and N/D method

In this section, we present the framework of the chiral unitary approach for s-wave meson-baryon scattering. We first discuss the scattering problem in a single channel for simplicity. Generalization to the coupled-channel scattering will be given in Sec. V. We consider the scattering of a pseudoscalar meson with mass m from a target baryon with mass M_T. The s-channel two-body unitarity condition for the amplitude $T(\sqrt{s})$ can be expressed as

$$\mathrm{Im}T^{-1}(\sqrt{s}) = \frac{\rho(\sqrt{s})}{2}, \qquad (1)$$

where $\rho(\sqrt{s}) = 2M_T\bar{q}/(4\pi\sqrt{s})$ is the two-body phase space of the scattering system with $\bar{q} = \sqrt{[s - (M_T - m)^2][s - (M_T + m)^2]}/(2\sqrt{s})$. This is the so-called elastic unitarity. Based on the N/D method [11], the general form of the scattering amplitude satisfying Eq. (1) is given by

$$T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s})}, \qquad (2)$$

where $V(\sqrt{s})$ is a real function expressing dynamical contributions other than the s-channel unitarity and will be

identified as the kernel interaction. $G(\sqrt{s})$ is obtained by the once subtracted dispersion relation with the phase-space function $\rho(\sqrt{s})$:

$$G(\sqrt{s}) = -\tilde{a}(s_0) - \frac{1}{2\pi}\int_{s^+}^{\infty} ds' \left(\frac{\rho(s')}{s'-s-i\epsilon} - \frac{\rho(s')}{s'-s_0}\right),\tag{3}$$

where $s^+ = (M_T + m)^2$ is the value of s at the s-channel threshold, $\tilde{a}(s_0)$ is the subtraction constant at the subtraction point s_0. One can easily verify that the amplitude given in Eqs. (2) and (3) satisfies Eq. (1).

Equivalently, the function $G(\sqrt{s})$ can be written as the finite part of the meson-baryon loop function

$$i\int \frac{d^4q}{(2\pi)^4}\frac{2M_T}{(P-q)^2 - M_T^2 + i\epsilon}\frac{1}{q^2 - m^2 + i\epsilon},\tag{4}$$

which is logarithmically divergent. Utilizing the dimensional regularization, we obtain the same structure as Eq. (3) up to a constant

$$\begin{aligned}G(\sqrt{s}) =& \frac{2M_T}{(4\pi)^2}\Big\{a(\mu) + \ln\frac{M_T^2}{\mu^2} + \frac{m^2 - M_T^2 + s}{2s}\ln\frac{m^2}{M_T^2}\\ &+ \frac{\bar{q}}{\sqrt{s}}[\ln(s-(M_T^2-m^2)+2\sqrt{s}\bar{q})\\ &+ \ln(s+(M_T^2-m^2)+2\sqrt{s}\bar{q})\\ &- \ln(-s+(M_T^2-m^2)+2\sqrt{s}\bar{q})\\ &- \ln(-s-(M_T^2-m^2)+2\sqrt{s}\bar{q})]\Big\},\end{aligned}\tag{5}$$

where $a(\mu)$ is the subtraction constant determined at the renormalization scale μ. The equivalence is verified by noting that both Eqs. (3) and (5) have the same imaginary part and that the real part satisfies the dispersion relation. For a single channel, there is only one degree of freedom for the regularization. Here we set $\mu = M_T$ from now on and simply denote the subtraction constant $a \equiv a(M_T)$, which plays the role of the ultraviolet cutoff parameter of the loop integral. A different choice of μ shifts a by a constant value without affecting the physics.

B. Kernel interaction

Let us consider the meaning of the function $V(\sqrt{s})$, which governs the dynamics of the system. In principle, $V(\sqrt{s})$ can be constructed once all the singularities on the complex energy plane are known. In practice, it is not possible, and we determine it with the help of chiral symmetry.

Regarding the $G(\sqrt{s})$ function as the meson-baryon loop function, we can interpret $T(\sqrt{s})$ in Eq. (2) as the solution of the Bethe-Salpeter equation with the kernel interaction $V(\sqrt{s})$. In the chiral unitary approach, it was shown that the off-shell effects can be absorbed into the renormalization of the kernel interaction [9, 15], leading to the algebraic solution given in Eq. (2), which

includes the resummation of the s-channel bubble diagrams. One way to determine the interaction kernel $V(\sqrt{s})$ is to match the unitarized amplitude $T(\sqrt{s})$ with the chiral perturbation theory order by order [11]. At leading order, where loops are absent, $V(\sqrt{s})$ is given by the s-wave interaction of the Weinberg-Tomozawa (WT) term [6, 7]

$$\begin{aligned}V(\sqrt{s}) =& V_{\mathrm{WT}}(\sqrt{s})\\ =& -\frac{C}{2f^2}[\sqrt{s}-M_T] \sim -\frac{C}{2f^2}\omega,\end{aligned}\tag{6}$$

where C is the group theoretical factor whose general form is given in Ref. [26], and f and ω are the decay constant and the energy of the meson, respectively. Based on the matching with the chiral perturbation theory, one can introduce higher order terms in $V(\sqrt{s})$ systematically [12, 39, 40, 41, 42, 43].

Here we note that if the CDD pole contribution exists, it should be included in $V(\sqrt{s})$ except for the pole at infinity which can be included in the subtraction constant. This is the prescription of the N/D method [29]. The effect of the CDD pole can be introduced by explicitly adding a resonance propagator in the interaction $V(\sqrt{s})$, in such a way that it does not violate the low-energy theorem [33]. While the higher order terms of the chiral Lagrangian may contain the CDD pole contribution implicitly [5, 35], the leading order WT term (6) is apparently not affected by the s-channel resonance structure [44].

C. Properties of the loop function

For later convenience, we now recall the general properties of the loop function. The loop function $G(\sqrt{s})$ is monotonically decreasing in the energy region below the threshold $\sqrt{s} \leq M_T + m$ [25, 26]. One can verify it by differentiating the expression in Eq. (3) with respect to $\sqrt{s}$:

$$\frac{dG}{d\sqrt{s}} = -\frac{1}{2\pi}\int_{s^+}^{\infty} ds'\frac{\rho(s')\sqrt{s}}{(s'-s+i\epsilon)^2},\tag{7}$$

which is negative for $(0\leq)\sqrt{s} \leq M_T + m$.

The physical s-channel scattering takes place above the threshold $\sqrt{s} \geq M_T + m$, which is on the unitarity (right hand) cut. The energy region below the threshold $\sqrt{s} \leq M_T + m$ corresponds to the bound state region of the s-channel scattering. In the present formulation of the N/D method, we fully take into account the unitarity cut, while the contribution from the unphysical (left hand) cut is included through order-by-order matching. This means that the crossed diagram in the u channel is treated only perturbatively. Our amplitude in Eq. (2) therefore should not be extrapolated to the energy region below the mass of the target $\sqrt{s} \leq M_T$, where the contributions from the u-channel diagrams become important.

As for the renormalization procedure of the loop function in Eq. (4), one can equivalently utilize procedures other than the dimensional regularization, such as the three-momentum cutoff scheme. On the one hand, the cutoff scheme provides an intuitive interpretation of the loop function in connection with the second-order perturbation of quantum mechanics. On the other hand, the dimensional regularization is compatible with the analyticity of the amplitude, which is suitable for the N/D method based on dispersion theory. We will make use of both renormalization schemes for the loop function in the following sections.

III. NATURAL RENORMALIZATION CONDITION

In this section, we propose the "natural renormalization scheme," which provides a suitable description for meson-baryon scattering without the CDD pole contribution. Our strategy is to determine theoretically the subtraction constant of the loop function in order to study the structure of the resonances. This is in contrast to the previous studies in which the subtraction constant is fitted to data. To determine the value of the subtraction constant theoretically, throughout this section, we assume that there is no contribution to the intermediate states in the loop function from the CDD pole and the amplitude follows the low-energy structure required by chiral symmetry. For illustration, the interaction kernel $V(\sqrt{s})$ is chosen to be the WT term $V_{\mathrm{WT}}(\sqrt{s})$ given in Eq. (6), which does not contain the CDD pole contribution. We may also consider higher order terms, such as quark mass terms. In this case, however, some of the higher order terms are known to contain resonance contributions. For the loop integral, we first show that the subtraction constant has an upper limit for the consistency with the physical interpretation of the loop function, which is inferred by familiar quantum mechanical problems. Next we consider the matching of the unitarized amplitude with the low-energy interaction, and we derive the allowed region of the subtraction constant. Combining these two conditions, we determine the natural subtraction constant for the dynamical generation of resonances in a way consistent with low-energy chiral dynamics. Note that this natural renormalization scheme is not aimed to describe an arbitrary meson-baryon scattering, but it assumes the absence of the CDD pole contribution in the loop function, as we discuss in detail below.

A. Consistency with physical loop function

Let us first consider the sign of the loop function (5) below the threshold $\sqrt{s} \leq M_T + m$ where the imaginary part vanishes. In the meson-baryon picture, we can assume that there are no states below the threshold contribut-

ing to the loop function as intermediate states. This sets up the model space of solving the scattering equation. In this case, the loop function should be negative below the threshold. This is essentially the same as what happens in the perturbative calculations of the energy of the lowest state which couples to higher states in a quantum mechanical system, where the energy correction becomes always negative.

This condition is automatically satisfied in the cutoff regularization; if we introduce a three-momentum cutoff q_{max}, the loop function can be written as

$$
\begin{aligned}
G^{3d}(\sqrt{s}) = &\frac{2M_T}{(2\pi)^2} \int_0^{q_{\mathrm{max}}} dq \frac{q^2}{E} \frac{1}{\omega} \\
&\times \frac{E + \omega}{(\sqrt{s} - (E + \omega) + i\epsilon)(\sqrt{s} + E + \omega)},
\end{aligned}
$$

with

$$
E = \sqrt{M_T^2 + q^2}, \quad \omega = \sqrt{m^2 + q^2}.
$$

This is always negative for $\sqrt{s} \leq M_T + m \leq E + \omega$, irrespective of the cutoff momentum q_{max}.

In the dimensional regularization, however, the real part can become positive if one takes a large positive value for the subtraction constant a in Eq. (5). This can be avoided by introducing an upper limit for the subtraction constant. As we discussed in the previous section, our amplitude can be in principle extrapolated down to $\sqrt{s} = M_T$. Since the loop function below the threshold is a decreasing function as seen in Eq. (7), to make the loop function negative for the relevant energy region $\sqrt{s} \geq M_T$, it is sufficient for $G(\sqrt{s})$ to have the negative value at $\sqrt{s} = M_T$, that is,

$$
G(M_T) \leq 0,
$$

which is equivalent to

$$
\begin{aligned}
a \leq a_{\mathrm{max}}^{(1)} = -\Big\{ &\frac{m^2}{2M_T^2} \ln \frac{m^2}{M_T^2} + \frac{m\sqrt{m^2 - 4M_T^2}}{2M_T^2} \\
&\times [\ln(m^2 + m\sqrt{m^2 - 4M_T^2}) \\
&+ \ln(2M_T^2 - m^2 + m\sqrt{m^2 - 4M_T^2}) \\
&- \ln(-m^2 + m\sqrt{m^2 - 4M_T^2}) \\
&- \ln(-2M_T^2 + m^2 + m\sqrt{m^2 - 4M_T^2})] \Big\}. \quad (8)
\end{aligned}
$$

If the subtraction constant satisfies this condition, the loop function with dimensional regularization is consistent with the physical requirement in the region of the s-channel scattering $(M_T \leq \sqrt{s})$.

B. Matching with the low-energy interaction

Next we require the amplitude $T(\sqrt{s})$ to follow the chiral low-energy theorem [45, 46, 47]. As a result of

the spontaneous chiral symmetry breaking, the scattering amplitude $T(\sqrt{s})$ can be expanded in powers of momenta of the pseudoscalar meson at low energy. Since we choose $V_{\mathrm{WT}}(\sqrt{s})$ for the interaction kernel as the leading order term of the chiral perturbation theory, the consistency of the low-energy theorem can be achieved by matching the full scattering amplitude $T(\sqrt{s})$ with the kernel interaction $V_{\mathrm{WT}}(\sqrt{s})$ at a certain scale $\sqrt{s} = \mu_m$:

$$T(\mu_m) = V_{\mathrm{WT}}(\mu_m), \tag{9}$$

which is realized when

$$G(\mu_m) = 0, \tag{10}$$

as easily seen in Eq. (2). Since the subtraction constant is a real number, Eq. (10) should be satisfied below the threshold $\mu_m \leq M_T + m$, otherwise the loop function has an imaginary part. On the other hand, the matching scale should not be far below the threshold, since the u-channel cut lies in the region $\sqrt{s} \leq M_T - m$, and the effect of the crossing dynamics becomes important at lower energies. Therefore, here we set the lower limit of the matching scale at $\mu_m = M_T$, to satisfy Eq. (9) within the s-channel scattering region. In summary, we impose the matching scale to lie in the region

$$M_T \leq \mu_m \leq M_T + m, \tag{11}$$

which corresponds to choosing the subtraction constant as

$$a_{\min}^{(2)} \leq a \leq a_{\max}^{(2)}, \tag{12}$$

with

$$a_{\min}^{(2)} = a_{\max}^{(1)}, \quad a_{\max}^{(2)} = -\frac{m}{M_T + m} \ln \frac{m^2}{M_T^2}.$$

The matching condition of Eq. (9) was discussed for πN scattering in Ref. [48]. It is reasonable to set the matching scale μ_m in this region when respecting the low-energy expansion. We note that for on-shell kinematics, the three-momentum is zero ($p = 0$) at $\sqrt{s} = M_T + m$, while it takes an imaginary value for the vanishing energy of the Nambu-Goldstone boson ($\omega = 0$) where $\sqrt{s} = \sqrt{M_T^2 - m^2} \sim M_T$. Since the chiral perturbation theory is valid for small four-momentum $p_\mu = (\omega, p)$, the matching scale μ_m should lie around the region (11). In the chiral limit $m \to 0$, the range (11) reduces into one value $\mu_m = M_T$, where $\omega = |p| = 0$.

One may consider the correction to Eq. (9) from the higher order interaction terms, such as the explicit symmetry breaking (quark masses) terms. Once again, our aim is to determine the property of the loop function by excluding the CDD pole from it. Therefore, in this case we can match the amplitude to the interaction V including the higher order corrections by the same condition $G(\mu_m) = 0$. The inclusion of the higher order terms in the interaction does not change the values of the subtraction constant in Eq. (12). In this case, however, we should note the possibility of having the CDD pole contribution in the interaction kernel V from the higher order chiral Lagrangian.

C. Natural value for the subtraction constant

Based on the physical meaning of the loop function and matching with the chiral amplitude at low energy, we have derived two conditions for the subtraction constant, Eqs. (8) and (12), which read $a_{\min}^{(2)} \leq a \leq a_{\max}^{(1)}$ with $a_{\min}^{(2)} = a_{\max}^{(1)}$. This means that the subtraction constant a_{natural} which satisfies both conditions is uniquely fixed by

$$a_{\mathrm{natural}} = a_{\min}^{(2)} = a_{\max}^{(1)}. \tag{13}$$

In terms of the zero of the loop function, this condition is equivalent to requiring

$$G(\mu_m) = 0, \quad \mu_m = M_T. \tag{14}$$

This subtraction constant is compatible with the absence of the CDD pole in the loop function, as we will discuss below. It also guarantees the matching of the scattering amplitude with the chiral low-energy interaction. We note once again that the subtraction constant so obtained does not necessarily explain experimental data. We have just specified a standard value of the subtraction constant. The relation to the phenomenologically determined value will be discussed in the next section.

In this renormalization condition, we exclude any states below the threshold as a model space of the unitarization, so that the unitarized amplitude in Eq. (2) naturally implements the meson-baryon picture in the model-building. Therefore, with the value of a_{natural}, the loop function does not include the CDD pole contribution. The condition (14) was already proposed in a different context in Ref. [12], where the matching with the u-channel scattering amplitude was emphasized. A similar argument with the present context based on chiral symmetry was given in Refs. [33, 48]. Our point is to regard this condition as the exclusion of the CDD pole in the loop function, based on the consistency with the negativeness of the loop function. For illustration, the loop function of the $\bar{K}N$ channel with $a = a_{\mathrm{natural}}$ is plotted in Fig. 1, where $M_T = 939$ MeV, $m = 496$ MeV, and $f = 106.95$ MeV are used.

Let us make some remarks on related works. First, in Ref. [11], a "natural" value for the subtraction constant was estimated to be $a \sim -2$, by comparing the loop function of dimensional regularization with that of three-momentum cutoff of ~ 630 MeV. This is different from our value of a_{natural}, practically and conceptually. In the present context, the value (13) is derived for the loop function as unaffected by CDD poles. We used the expression of the three-momentum cutoff to illustrate that the real part of the loop function is negative below threshold. It is not needed to introduce the explicit scale (such as ~ 630 MeV) of the cutoff in our case. The "natural" value in Ref. [11] can, in principle, be applied to any system, as long as the typical cutoff scale of the physics is around ~ 630 MeV. On the other hand, our
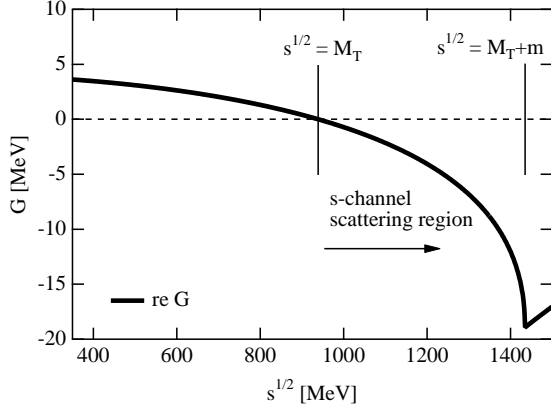
FIG. 1: Real part of the loop function $G(\sqrt{s})$ for the $\bar{K}N$ channel when $a = a_{\text{natural}}$ is adopted. The s-channel scattering region is defined as $\sqrt{s} > M_T$.

natural renormalization scheme is introduced just for excluding the CDD poles; it does not describe the scattering with CDD poles. This possibility is considered in the next section. Second, we introduce the condition of matching (9) to determine the value of the subtraction constant explicitly, along the same line with Ref. [48]. This is different from the order-by-order matching introduced in Ref. [11]. The latter is a conceptual matching used to derive the form of the interaction kernel V. Our condition (14) explicitly require the vanishing of the loop function at a certain low-energy point. Then the value of the subtraction constant is determined, once again, for the loop function without CDD poles.

IV. INTERPRETATION OF PHENOMENOLOGICAL MODEL

In this section, we discuss the origin of the dynamically generated resonances by reanalyzing the simplest phenomenological model to determine the subtraction constant in the chiral unitary approach, in comparison with the natural renormalization scheme. Let us assume that we have enough experimental data for the system of interest from the low-energy to the resonance-energy region. From the viewpoint of the renormalization, once the scattering amplitude T (observable) is fixed, the change of the renormalization parameter in the loop function G can be absorbed into the change of the interaction V. In other words, we cannot determine *a priori* the interaction kernel and the loop function separately. Thus, for a given amplitude T, we can construct different sets of interaction V and loop function G,

$$T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s};a) - G(\sqrt{s};a)}, \qquad (15)$$

where a labels the renormalization scheme. Once we specify either the loop function G or the interaction ker-

nel V, we also determine the other one by Eq. (15) to reproduce the same amplitude T.

In the conventional phenomenological approaches, the interaction kernel V is determined in the beginning by chiral perturbation theory. For instance, in the simplest models, the interaction kernel V is chosen to be the leading order WT term,

$$T(\sqrt{s}) = \frac{1}{V_{\text{WT}}^{-1}(\sqrt{s}) - G(\sqrt{s};a_{\text{pheno}})}, \qquad (16)$$

with the subtraction constant a_{pheno} in the loop function G being a free parameter to reproduce experimental data. We call this procedure the phenomenological renormalization scheme. This scheme can describe various phenomena well, but the subtraction constant does not always satisfy the natural renormalization condition in Eq. (13). Such a subtraction constant takes care of the contributions that are not included in the interaction kernel V_{WT}.

The renormalization condition proposed in the previous section is to fix the subtraction constant such that in the resulting loop function there is no contribution from states below the threshold. To achieve the equivalent scattering amplitude, we need to adopt a different interaction kernel V_{natural} as

$$T(\sqrt{s}) = \frac{1}{V_{\text{natural}}^{-1}(\sqrt{s}) - G(\sqrt{s};a_{\text{natural}})}, \qquad (17)$$

with the subtraction constant a_{natural} given in Eq. (13). The interaction kernel V_{natural} should be modified from V_{WT} in order to reproduce experimental observables.

The physical observable T should equivalently be reproduced by both renormalization schemes. Thus, equating the denominators of Eqs. (16) and (17)

$$V_{\text{natural}}^{-1}(\sqrt{s}) - G(\sqrt{s};a_{\text{natural}})$$
$$= V_{\text{WT}}^{-1}(\sqrt{s}) - G(\sqrt{s};a_{\text{pheno}}) , \qquad (18)$$

we obtain the interaction kernel V_{natural} in the natural renormalization scheme as

$$V_{\text{natural}}^{-1}(\sqrt{s}) = V_{\text{WT}}^{-1}(\sqrt{s}) - \frac{2M_T}{16\pi^2}\Delta a , \qquad (19)$$

with $\Delta a \equiv a_{\text{pheno}} - a_{\text{natural}}$. Here we have exploited the fact that the dependence of a in the loop function G reads constant shift, as seen in Eq. (5). Using the explicit form of the WT term (6), we finally obtain the interaction kernel in the natural renormalization condition as

$$V_{\text{natural}}(\sqrt{s}) = \frac{1}{-\frac{2f^2}{C(\sqrt{s}-M_T)} - \frac{2M_T\Delta a}{16\pi^2}} \qquad (20)$$

$$= -\frac{8\pi^2}{M_T\Delta a}\frac{\sqrt{s}-M_T}{\sqrt{s}-M_{\text{eff}}}, \qquad (21)$$

with an effective mass

$$M_{\text{eff}} \equiv M_T - \frac{16\pi^2 f^2}{CM_T\Delta a}. \qquad (22)$$

Hereafter, we call $V_{\text{natural}}(\sqrt{s})$ the effective interaction in the natural renormalization scheme. The expression in Eq. (21) tells us that the interaction kernel $V_{\text{natural}}(\sqrt{s})$ can have a pole, which lies in the s-channel scattering region with an attractive interaction $C > 0$ and a negative value for Δa. Extracting the WT term from the effective interaction (21), we find

$$
\begin{aligned}
V_{\text{natural}}(\sqrt{s}) = & - \frac{C}{2f^2}(\sqrt{s} - M_T) + \frac{C}{2f^2}\frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\text{eff}}} \\
\equiv & V_{\text{WT}}(\sqrt{s}) + \Delta V(\sqrt{s}; \Delta a). \quad (23)
\end{aligned}
$$

The second term can be interpreted as the pole whose energy dependence is consistent with the chiral expansion, since the pole term is quadratic in powers of the meson energy $\omega = \sqrt{s} - M_T$, while the leading WT term is linear in it. This is consistent with the schematic discussion made in Refs. [42, 43] that the change of the subtraction constant may introduce the effect of the higher order terms in the kernel interaction. Mathematically, it is also possible to have a pole for a repulsive interaction $C < 0$ with $\Delta a > 0$. If the experiments require such a value for the phenomenological subtraction constant, the effective interaction would be the repulsive contact interaction plus an explicit resonance term. The $\pi\pi$ scattering amplitude in the linear σ model is an example of this case [49].

The relevance of the second term of Eq. (23) depends on the scale of the effective mass M_{eff}, which is obtained by the difference of the phenomenological and natural subtraction constants Δa. If Δa is small, the effective pole mass M_{eff} becomes large. In this case, the second term of Eq. (23) can be neglected or gives smooth energy dependence in the resonance energy region $\sqrt{s} \sim M_T + m \ll M_{\text{eff}}$. If the difference Δa is large, the effective mass M_{eff} gets closer to the threshold. In this case, the pole contribution is no longer negligible. This means that the use of a negative Δa with large absolute value is equivalent to the introduction of a pole in the chiral Lagrangian. We therefore consider that the pole in the effective interaction (21) is a source of the physical resonances in this case. It was known that the higher order term could be a source of a resonance in the full amplitude, because these terms behave as the contracted resonance propagator in the s channel. Here we point out a possible source of the resonance in the conventional chiral unitary model, even if we use the leading order chiral interaction.

At this stage, two renormalization schemes (16) and (17) are interpreted as follows. In the phenomenological scheme (16), the interaction kernel V_{WT} does not include the CDD pole contribution, while in the natural scheme (17) the loop function G does not contain the CDD pole, as discussed in the previous section. Therefore, when the physical amplitude contains the CDD pole contribution, the effect is attributed to $G(\sqrt{s}; a_{\text{pheno}})$ in the phenomenological scheme, while to $V_{\text{natural}}(\sqrt{s})$ in the natural scheme. Indeed, we have demonstrated that

$V_{\text{natural}}(\sqrt{s})$ contains a resonance propagator. In the limit $\Delta a \to 0$, the two schemes agree with each other, which corresponds to the amplitude compatible with the meson-baryon picture of resonances, as explained in Sec. III. Note also that in the N/D method, the CDD pole contributions except for those at infinity should be included in the interaction kernel V, since the loop function G expresses the only contribution from the unitarity cut. In this respect, the phenomenological scheme has accommodated the CDD pole contribution in the loop function. In contrast, the natural scheme has more similarity to the formulation of the N/D method, as the CDD pole contribution is explicitly seen in the interaction kernel.

It is worth noting that the energy dependence of the interaction $V_{\text{WT}}(\sqrt{s})$ leads to the pole in the effective interaction, since the effective pole mass M_{eff} is obtained by solving the equation

$$
1 - A \cdot V_{\text{WT}}(\sqrt{s}) = 0, \quad A = \frac{2M_T \Delta a}{16\pi^2}.
$$

Thus, for an energy-independent interaction V, no pole can appear. Taking into account that the coupling should be a derivative type in the nonlinear realization of chiral symmetry, the mechanism can be applied to any unitarized model with chiral interaction, such as σ and ρ mesons in the meson-meson scattering.

The interaction kernel in the natural renormalization scheme $V_{\text{natural}}(\sqrt{s})$ can also be expressed by renormalizing Δa to an effective coupling strength f':

$$
V_{\text{natural}}(\sqrt{s}) \equiv -\frac{C}{2(f')^2}(\sqrt{s} - M_T), \quad (24)
$$

where the change of the coupling strength is then given by

$$
(f')^2 - f^2 = \frac{CM_T \Delta a}{16\pi^2}(\sqrt{s} - M_T).
$$

In the region of s-channel scattering $\sqrt{s} > M_T$ for attractive interaction $C > 0$, we find that positive Δa increases f^2 (and the interaction becomes less attractive), and negative Δa decreases f^2 (more attractive). In this way, we can translate the change of the subtraction constant into the change of the strength of the interaction kernel. This is again consistent with the argument in Refs. [42, 43].

As we mentioned, $G(\sqrt{s})$ is monotonically decreasing for $\sqrt{s} \leq M_T + m$. Since the subtraction constant a appears with a positive sign in $G(\sqrt{s})$, we find that positive (negative) Δa makes μ_m increase (decrease). In this respect, the renormalization condition $\mu_m = M_T$ adopted in Refs. [25, 26] was the most advantageous prescription to generating a bound state, with the matching scale being in the s-channel scattering region $\mu_m \geq M_T$ [45, 46, 47].

V. GENERALIZATION TO THE COUPLED-CHANNEL SCATTERING

The arguments given so far can be applied to the meson-baryon scattering in the flavor-symmetric limit, where channel couplings are absent. In practice, the physically interesting system is not flavor symmetric; that is, the physical masses for particles break the flavor symmetry. As a consequence, we encounter a coupled-channel scattering problem in the chiral unitary approach. The interaction and amplitude are extended to matrix forms with channel indices $V_{ij}(\sqrt{s})$ and $T_{ij}(\sqrt{s})$, and the scattering equation (2) is expressed as a matrix equation. The loop function is given by a diagonal matrix whose ith component is given by $G_i(\sqrt{s})$, with a different threshold for each channel i. The generalization of the natural renormalization scheme (13) or (14) to the coupled-channel case is straightforward, once the differences of the thresholds and the masses of the target hadron M_i are properly taken into account. For an illustration of the following argument, we show the plot of the mass of the target baryon M_i and the threshold $M_i + m_i$ for $S = -1$ and $I = 0$ meson-baryon scattering in Fig. 2.

A. Natural values for the subtraction constants in coupled-channel scattering

We first note that the matching $T_{ij}(\mu_m) = V_{ij}(\mu_m)$ in matrix form can be achieved when the loop functions in *all* channels are zero at a common scale μ_m:

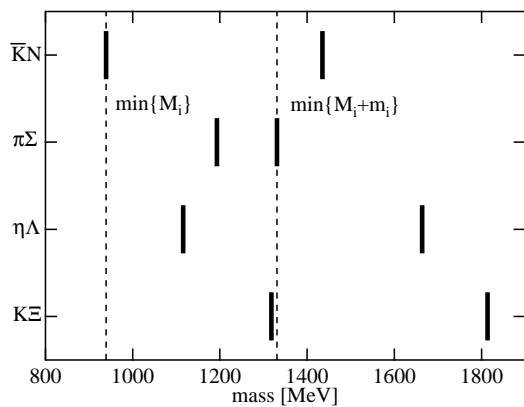$$G_i(\mu_m) = 0. \qquad (25)$$

FIG. 2: Masses of the target baryons M_i and threshold energies $M_i + m_i$ of channel i for $S = -1$ and $I = 0$ meson-baryon scattering. The dashed line on the left denotes the lowest mass of the target hadron ($\min\{M_i\}$), the mass of the nucleon; the dashed line on the right stands for the lowest threshold energy of the $\pi\Sigma$ ($\min\{M_i + m_i\}$).

This equation can be achieved when the imaginary parts of all the loop functions vanish, namely, below the lowest threshold:

$$\mu_m \leq \min\{M_i + m_i\}. \qquad (26)$$

Recalling the discussion in Sec. III, Eq. (25) should be imposed in the s-channel scattering region in order to satisfy the consistency with the physical loop function (Sec. III A) and the matching of the full amplitude to the low-energy interaction (Sec. III B). In the coupled-channel case, however, the meaning of the "s-channel scattering region" is not clear, since masses of target baryons M_i depend on their channel i. Here we propose a way to fix the scale μ_m by

$$G_i(\mu_m) = 0, \quad \mu_m = \min\{M_i\}, \qquad (27)$$

in the s-channel regions for all the channels. We adopt this condition for the natural renormalization scheme in the coupled-channel scattering. The natural values for the subtraction constants $a_{\text{natural},i}$ can be determined such that the loop function satisfies the condition (27). With this condition, the loop functions in all channels are negative for their s-channel scattering region, and the full amplitude $T_{ij}(\sqrt{s})$ reduces to the tree level one at $\sqrt{s} = \min\{M_i\}$. The scale $\mu_m = \min\{M_i\}$ lies in the u-channel region for channels with $M_i > \mu_m$, but the extrapolation is of the order of mass difference of the particles, which is coming from the flavor-symmetry breaking and therefore is not very large.

The condition (27) is one of the ways to achieve the natural renormalization in the coupled-channel cases. In principle, we have other options for μ_m that satisfy Eq. (25). For instance, in Ref. [12], μ_m is taken to be the mass of the hadron of the same strangeness as the scattering system, i.e., $\mu_m = m_\Lambda$ for the $S = -1$ and $I = 0$ channels. Here we put more weight on the consistency with the physical loop function $[G(\mu_m) \leq 0]$ and choose the lowest mass of the target hadrons with $\mu_m = m_N$.

B. Effective interaction in coupled-channel scattering

Once the experimental amplitudes are fitted by phenomenological models with $a_{\text{pheno},i}$, we can interpret the origin of the resonances in a manner similar to that in Sec. IV. The WT term in the coupled-channel case is given by[2]

$$V_{\text{WT},ij}(\sqrt{s}) = -\frac{C_{ij}}{4f^2}[2\sqrt{s} - M_i - M_j], \qquad (28)$$

---

[2] Here we ignore the small factor $\sqrt{(M_i + E_i)(M_j + E_j)/(4M_i M_j)}$ for simplicity of the discussion of the poles. In the numerical analysis in Sec. VI, we include this factor, although the quantitative effect is small: deviation of pole positions of the scattering amplitude is less than 1 MeV.

with the coupling matrix C_{ij} fixed by the SU(3) group structure of the channels. The equation for the amplitude (15) should be read as a matrix equation. Comparing the phenomenological and natural schemes, the effective interaction in the natural renormalization scheme is found to be

$$V_{\mathrm{natural}}(\sqrt{s}) = \left(V_{\mathrm{WT}}^{-1}(\sqrt{s}) - A\right)^{-1}, \qquad (29)$$

with a diagonal matrix

$$A_{ij} = \frac{2M_i \Delta a_i}{16\pi^2} \delta_{ij}, \quad \Delta a_i = a_{\mathrm{pheno},i} - a_{\mathrm{natural},i}.$$

Because Eq. (29) is a matrix equation, Δa_i in channel i affects the interactions in all channels. To discuss the poles in the effective interaction, we rewrite it as

$$\begin{aligned} V_{\mathrm{natural}}(\sqrt{s}) =& V_{\mathrm{WT}}(\sqrt{s}) \left(\mathbf{1} - A \cdot V_{\mathrm{WT}}(\sqrt{s})\right)^{-1} \\ =& V_{\mathrm{WT}}(\sqrt{s}) \frac{1}{\det\left[\mathbf{1} - A \cdot V_{\mathrm{WT}}(\sqrt{s})\right]} \\ & \times \mathrm{cof}\left[\mathbf{1} - A \cdot V_{\mathrm{WT}}(\sqrt{s})\right], \end{aligned}$$

where $\det X$ and $\mathrm{cof}\, X$ are the determinant and the cofactor matrix of X. The poles in the effective interaction are then obtained by

$$\det\left[\mathbf{1} - A \cdot V_{\mathrm{WT}}(\sqrt{s})\right] = 0. \qquad (30)$$

As seen in Eq. (28), each component of $V_{\mathrm{WT}}(\sqrt{s})$ is given by the linear function of $\sqrt{s}$, so Eq. (30) is an nth order algebraic equation of $\sqrt{s}$ for the n-channel problem. There are n roots for Eq. (30), $z_i (i = 1, \ldots, n)$ which correspond to poles in the effective interaction. It is also possible to have a pair of complex poles which are conjugate of each other. We interpret the imaginary part of the pole as the width of the pole in the effective interaction, although there is no information of the threshold in the construction of the effective interaction. For the number of channels smaller than 5, the pole positions of the effective interaction can be obtained by analytically solving Eq. (30).

In the coupled-channel case, around the energy region close to a pole position z_{eff}, the effective interaction can be expressed as

$$V_{\mathrm{natural},ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - z_{\mathrm{eff}}},$$

where g_i is the coupling strength to channel i, which is a complex number in general. We can extract g_i from the residue of the pole:

$$\begin{aligned} R_{ij} =& (\sqrt{s} - z_{\mathrm{eff}}) V_{\mathrm{natural},ij}(\sqrt{s})|_{\sqrt{s}=z_{\mathrm{eff}}} \\ =& g_i g_j. \end{aligned} \qquad (31)$$

When we know all the roots of Eq. (30), residues of the pole z_{eff} can be calculated analytically as

$$\begin{aligned} R_{ij} =& V_{\mathrm{WT},im}(z_{\mathrm{eff}}) \frac{(4f^2)^n}{2^n \det[A \cdot C] \Pi_{l \neq \mathrm{eff}}^n (z_{\mathrm{eff}} - z_l)} \\ & \times \mathrm{cof}\left[\mathbf{1} - A \cdot V_{\mathrm{WT}}(z_{\mathrm{eff}})\right]_{mj}. \end{aligned}$$

TABLE I: Natural and phenomenological values [43] for the subtraction constants with the regularization scale $\mu = M_i$.

$S = -1$	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$a_{\mathrm{pheno},i}$	-1.042	-0.7228	-1.107	-1.194
$a_{\mathrm{natural},i}$	-1.150	-0.6995	-1.212	-1.138
$S = 0$	πN	ηN	$K\Lambda$	$K\Sigma$
$a_{\mathrm{pheno},i}$	1.509	-0.2920	1.454	-2.813
$a_{\mathrm{natural},i}$	-0.3976	-1.239	-1.143	-1.138

As in the single-channel case, we define the deviation of the interaction ΔV_{ij} as

$$V_{\mathrm{natural},ij}(\sqrt{s}) = V_{\mathrm{WT},ij}(\sqrt{s}) + \Delta V_{ij}(\sqrt{s}), \qquad (32)$$

from which we can estimate the effect of Δa_i by $\Delta V_{ij}(\sqrt{s})$.

VI. NUMERICAL ANALYSIS

By now we have established the natural renormalization scheme to interpret the origin of the poles found in the phenomenological models. In this section, we apply our method to physical meson-baryon scatterings in $S = -1$ and $I = 0$ channel and $S = 0$ and $I = 1/2$ channel, where the $\Lambda(1405)$ and $N(1535)$ resonances are generated, respectively. We use the isospin averaged masses for mesons and baryons, and $f = 106.95$ MeV. The coupling strength C_{ij} can be calculated by the general expression in Ref. [26], while the explicit numbers can be found in Refs. [38, 50]. For these channels, the scattering observables such as cross sections and phase shifts are well reproduced by the WT term together with the subtraction constants $a_{\mathrm{pheno},i}$ [42, 43], which are based on the results in Refs. [38, 50]. On the other hand, according to Eq. (27), we obtain the natural values of the subtraction constants $a_{\mathrm{natural},i}$ by setting $G(M_N) = 0$ for all channels.

Both $a_{\mathrm{pheno},i}$ and $a_{\mathrm{natural},i}$ are shown in Table I. At first glance, the phenomenological subtraction constants are similar to the natural values for $S = -1$ channels, while they are not so for $S = 0$ channels. This indicates that $\Lambda(1405)$ has a large component of a dynamically generated resonance of a meson-baryon system, but $N(1535)$ requires some contribution supplied by the subtraction constants, in addition to the dynamical meson-baryon component.

First of all, we show the pole positions for $\Lambda(1405)$ and $N(1535)$ in the amplitudes obtained by the phenomenological renormalization scheme. With the phenomenological subtraction constants $a_{\mathrm{pheno},i}$ and the WT interaction V_{WT}, we find pole positions at

$$z_1^{\Lambda^*} = 1429 - 14i \text{ MeV}, \quad z_2^{\Lambda^*} = 1397 - 73i \text{ MeV}, \quad (33)$$

for the $\Lambda(1405)$ in $S = -1$ scattering. Note that this resonance is expressed by two poles [51], which stem from
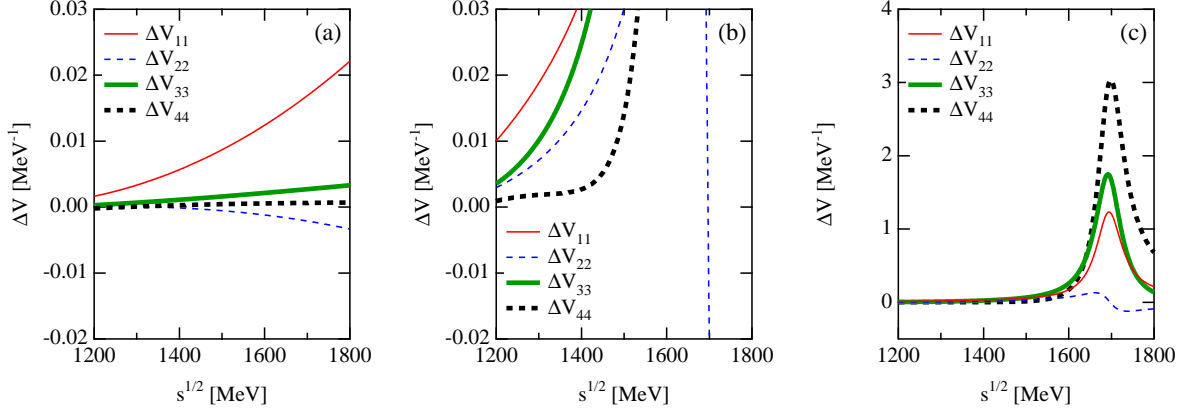
FIG. 3: (Color online) Deviations of the effective interactions from the Weinberg-Tomozawa term $\Delta V_{ii}(\sqrt{s})$ defined in Eq. (32), (a) $S = -1$ channels, (b) enlargement of panel (c), (c) $S = 0$ channels. The channels 1–4 correspond to $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$, and $K\Xi$ for $S = -1$ channels, and to πN, ηN, $K\Lambda$, and $K\Sigma$ for $S = 0$ channels, respectively.

TABLE II: Coupling strengths g_i of the pole in the effective interaction of the $S = 0$ channel [Eq. (35)].

	πN	ηN	$K\Lambda$	$K\Sigma$
g_i	$3.42 + 3.18i$	$-0.192 - 2.14i$	$-3.92 - 4.15i$	$5.99 + 4.42i$
$|g_i|$	4.67	2.15	5.71	7.44

the attractive forces in $\bar{K}N$ and $\pi\Sigma$ channels [52]. In the $S = 0$ scattering amplitude, a pole is found at

$$z^{N^*} = 1493 - 31i \text{ MeV,} \qquad (34)$$

which corresponds to $N(1535)$. These poles reproduce the properties of $\Lambda(1405)$ and $N(1535)$ as well as the scattering observables such as the total cross sections and the phase shifts [42, 43].

Next we evaluate the effective interaction in the natural renormalization scheme based on Eq. (29), and extract the deviation from the WT term as in Eq. (32). We plot the diagonal components of the deviation $\Delta V_{ii}(\sqrt{s})$ in Fig. 3. We observe that $\Delta V_{ii}(\sqrt{s})$ are small in the $S = -1$ channel case, whereas the deviations are large in the $S = 0$ channel in the relevant energy region of $1400 \leq \sqrt{s} \leq 1600$ MeV. Moreover, we observe a bump structure at around 1700 MeV in the $S = 0$ channel [Fig. 3(c)]. The origin of this structure is due to the poles found in the effective interaction at

$$z^{N^*}_{\text{eff}} = 1693 \pm 37i \text{ MeV.} \qquad (35)$$

These poles may contribute to the structure of the generated $N(1535)$ in the full amplitude whose pole is found at a similar energy as shown in Eq. (34). We also calculate the coupling strength g_i to each channel, which is obtained as the residue of the pole in the effective interaction as in Eq. (31). The values of the couplings are summarized in Table II. We observe that the pole strongly couples to the $K\Sigma$ channel. As seen in Table I,

the difference of the subtraction constants in the $K\Sigma$ channel has a large negative value, $\Delta a_{K\Sigma} = -1.67$. This indicates that the important ingredient for $N(1535)$ to be added to the WT interaction is in the $K\Sigma$ channel.

We estimate theoretical uncertainty of the pole location in Eq. (35) for $N(1535)$ within the coupled-channel natural renormalization scheme. Although we have chosen Eq. (27) as a condition for the natural renormalization, as we mentioned above, we may choose another matching scale within the region

$$G_i(\mu_m) = 0, \quad \min\{M_i\} \leq \mu_m \leq \min\{M_i + m_i\}, \quad (36)$$

in which, except for the original condition $\mu_m = \min\{M_i\}$, loop functions in some channels become positive at $\sqrt{s} > M_i$ with the order of the flavor-symmetry breaking. Depending on the choice of the natural renormalization condition, the values of a_{natural} change slightly. As a consequence, the pole positions, which are the solutions of Eq. (30), depend on a_{natural} through the matrix A_{ij}. Varying the matching scale between the upper and lower values of Eq. (36), we find the pole of the effective interaction in the region from $z^{N^*}_{\text{eff}} = 1693 \pm 37i$ to $z^{N^*}_{\text{eff}} = 1673 \pm 146i$ MeV. The pole in the effective interaction can be interpreted as a "bare state," which will be dressed by the meson-baryon cloud through the unitarization procedure. It is therefore expected that the pole in the physical amplitude of Eq. (34) evolves from one of the bare poles found here.

In general, the effective interaction contains n poles, since Eq. (30) has n roots. The relevant point is the energy scale of the pole position. If poles appear in the energy region of our interest, as in the case of $N(1535)$, the effect of the pole on the phenomenology is significant. On the other hand, if poles are located away from the physically resonant region, these poles are irrelevant to the physical observables. In this respect, it is instructive to evaluate the pole of the effective interaction for $\Lambda(1405)$. Calculating Eq. (30) for the $S = -1$ channel,

we find a pole with almost no imaginary part,

$$z_{\text{eff}}^{\Lambda^*} \sim 7.9 \text{ GeV}.$$

This is far from the relevant energy scale; therefore, the pole plays essentially no role for the $\Lambda(1405)$ physics of our interest. Even if the poles in the physical amplitude of Eq. (33) originates in this bare pole, a substantial effect from the meson-baryon dynamics would be required. Therefore $\Lambda(1405)$ is largely dominated by the component of the dynamical meson and baryon.

We also investigate the pole positions with the natural renormalization with the WT interaction to see effects of the dynamical component on the resonance. When we choose the natural values $a_{\text{natural},i}$, we find

$$z_1^{\Lambda^*} = 1417 - 19i \text{ MeV}, \quad z_2^{\Lambda^*} = 1402 - 72i \text{ MeV},$$

for $\Lambda(1405)$, and

$$z^{N^*} = 1582 - 61i \text{ MeV}, \tag{37}$$

for $N(1535)$.[3] We plot the pole positions in Fig. 4. The poles for $\Lambda(1405)$ are very similar to those obtained by the phenomenological subtraction constants. This again indicates the dominance of the meson-baryon component in $\Lambda(1405)$. On the other hand, the pole for $N(1535)$ moves to the higher energy when we use the natural values. Since a sizable attractive interaction exists, a pole can be generated in $S = 0$ scattering, although the amplitude is not in good agreement with experimental data, as indicated by the difference of the pole positions. For the theoretical ambiguity of the pole position of Eq. (37) in the natural renormalization, Eq. (36) leads to the pole position of the amplitude as

$$z^{N^*} \sim (1582\text{–}1602) \pm (61\text{–}65)i \text{ MeV}. \tag{38}$$

These results in different natural schemes are still far from the value in Eq. (34), with which the amplitude successfully reproduces experimental data.

Study of the coupling properties of the pole for $N(1535)$ is instructive to further understand the origin of the resonance. In Table III, we show the coupling strengths of the $N(1535)$ pole in the phenomenological amplitude of Eq. (34). In Ref. [37], the pole in the physical amplitude exhibited a similar coupling tendency with the pole in the effective interaction. Based on this observation, Ref. [37] concluded that the CDD pole contribution dominates in the physical state. In the present case,
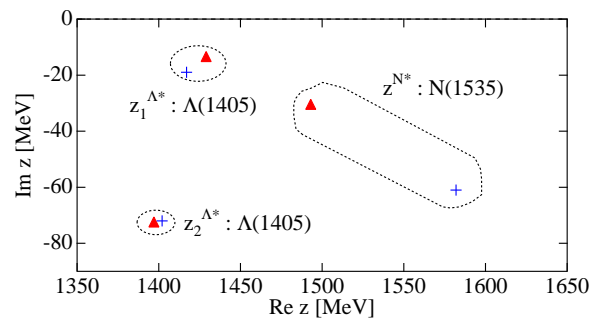

FIG. 4: (Color online) Pole positions of the meson-baryon scattering amplitudes. The triangles stand for the pole positions with the phenomenological amplitude; crosses denote the pole positions in the natural renormalization with the WT interaction. $z_1^{\Lambda^*}$ and $z_2^{\Lambda^*}$ are the poles for $\Lambda(1405)$ in the $S = -1$ scattering amplitude, and z^{N^*} is the pole for $N(1535)$ in the $S = 0$ amplitude.

comparing Table III with that in the effective interaction (Table II), we find that this is not the case for $N(1535)$ in the present model. On the other hand, the coupling property of the phenomenological $N(1535)$ is more similar to that of the pole in the amplitude by the WT term with the natural renormalization scheme [Eq. (37)] shown in Table IV. Since the latter is attributed to the meson-baryon dynamical component of the resonance, the analysis of the coupling strengths indicates the importance of the meson-baryon component in $N(1535)$ in addition to the CDD pole contribution.

In summary for the numerical analysis, we have studied the origin of $\Lambda(1405)$ and $N(1535)$ based on the natural renormalization scheme and phenomenological amplitude of the meson-baryon scattering. The $S = -1$ scattering and $\Lambda(1405)$ are well reproduced by the natural renormalization with the WT term, indicating that the $\Lambda(1405)$ resonance is a (mostly) pure dynamical resonance. In contrast, the $S = 0$ scattering and the $N(1535)$ resonance is not reproduced by the WT term only, and the translation of the phenomenological subtraction constants into the low-energy effective interaction requires a pole term of which the mass is around 1700 MeV in addition to the WT interaction. At the same time, the dynamical component is also important for the structure of $N(1535)$, since the attractive interaction of the WT term is strong enough to generate a resonance in the natural renormalization, and the coupling property of $N(1535)$ is closer to the dynamical resonance. Therefore, we interpret $N(1535)$ as mixture of a pole singularity of genuine state with the dynamical component.

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[3] In Ref. [13], they used the WT term with natural values of the subtraction constants were used, and found a pole at position similar to that in Eq. (34). We confirm their result with $f = 90$ MeV; however, the amplitude is not fitted to the scattering data. If we adopt the original model in Ref. [38], namely, by choosing the channel-dependent f_i, the pole position of the phenomenological model becomes $z^{N^*} = 1533 - 37i$ MeV, which is closer to the result with the natural scheme [Eq. (37)], but not in as good agreement as in the case of the $S = -1$ channel.

VII. DISCUSSION

The results of the present analysis can be argued in various theoretical perspectives. There are several discussions about the structure of the baryon resonances:

TABLE III: Coupling strengths g_i of the pole in the phenomenological amplitude of $S = 0$ channel [Eq. (34)].

	πN	ηN	$K\Lambda$	$K\Sigma$
g_i	$0.911 + 0.256i$	$1.60 - 0.374i$	$-1.40 - 0.393i$	$2.92 - 0.451i$
$|g_i|$	0.949	1.64	1.45	2.96

TABLE IV: Coupling strengths g_i of the pole in the natural renormalization of $S = 0$ channel [Eq. (37)].

	πN	ηN	$K\Lambda$	$K\Sigma$
g_i	$0.126 + 0.330i$	$-1.99 - 0.700i$	$-1.63 + 0.508i$	$-2.90 + 0.359i$
$|g_i|$	0.353	2.11	1.71	2.93

three-quark versus five-quark, or hadronic molecule versus quark originated structure. In principle, all these structures eventually stem from QCD dynamics and mix with each other. Nevertheless, it helps our physical understanding to extract several components out of a resonance state and inspect the dominant contribution to the resonance. For instance, the $\Lambda(1405)$ resonance can be schematically decomposed as [53]

$$
\begin{aligned}
|\Lambda(1405)\rangle = & N_{MB}|B\rangle|M\rangle + \cdots \\
& + N_3|qqq\rangle + N_5|qqqq\bar{q}\rangle + \cdots,
\end{aligned} \quad (39)
$$

where $|B\rangle|M\rangle$ is the dynamical meson-baryon component in the scattering theory of hadrons [54, 55]; and the rest, which corresponds to the CDD pole and is not represented by the meson-baryon state, is expanded by the number of quarks.

According to the decomposition (39), the present analysis for $\Lambda(1405)$ unveils a large weight of the N_{MB}. Probably, the best way to disentangle the dynamical component from the CDD pole contribution is the model-independent determination proposed in Ref. [56]. Unfortunately, the applicability of this method is limited, and it seems to be difficult to deal with the resonances considered in this paper [57]. Therefore, our analysis, though studied in a specific model, can be regarded as an alternative approach to this subject with larger applicability.

Another powerful method for clarifying the internal structure of the resonances is the use of the number of colors (N_c). It is well known that the only $\bar{q}q$ meson survives in the large N_c limit. The property of the meson resonances in the large N_c limit was studied in a dynamical approach [29]. It was found that the ρ meson survives in the large N_c limit while the σ disappears, indicating the $\bar{q}q$ nature of the former resonance. A systematic study of the N_c scaling of the resonance parameters around $N_c = 3$ was performed in Ref. [36], leading to the same conclusion for the properties of the mesonic resonances. In the context of the baryon resonance, the scaling behavior of the qqq baryon with N_c is known from the general argument, so it is possible to investigate whether the N_3 component dominates. The method of N_c scaling has been applied to $\Lambda(1405)$ in Ref. [58], where the N_c behavior of both poles for $\Lambda(1405)$ given in Eq. (33) indicates their non-qqq structure. Concerning $\Lambda(1405)$,

the present result (N_{MB} dominates) and the result in Ref. [58] ($N_3 \ll 1$) consistently imply that the $\Lambda(1405)$ resonance is dominated by the meson-baryon molecular component.

As for $N(1535)$, we have found substantial contribution other than those from N_{MB}. There is an interesting possibility of the origin of this CDD pole contribution: a chiral partner of the ground state nucleon. The chiral partner is a parity pair of the particles which transform each other under the linear realization of the chiral transformation and become degenerate when chiral symmetry is restored. Familiar candidates are (ρ, a_1) and (σ, π) in the meson sector. Since $N(1535)$ is the lowest negative-parity state having the same quantum number as the ground state nucleon, it is a candidate for the chiral partner of the nucleon [59, 60, 61, 62, 63]. In the linear realization of chiral symmetry, the chiral partner is introduced as an explicit field in the chiral symmetric Lagrangian. Such an explicit field is expressed as a CDD pole in the chiral unitary approach. Therefore, the CDD pole found here could be interpreted as the chiral partner of the nucleon.

On the other hand, as indicated by the coupling-strength analysis, the strong meson-baryon interaction in the $S = 0$ channel also provides a sizable meson-baryon component on top of the quark-originated $N(1535)$. In Ref. [64], electrotransition form factors of $N(1535)$, namely, the helicity amplitudes $A_{1/2}$ and $S_{1/2}$, have been discussed in the meson-baryon picture. There $N(1535)$ is expressed by the chiral unitary approach with the phenomenological renormalization scheme and the transition $\gamma^* N \to N(1535)$ was computed by considering the photon coupling only to the constituent meson and baryon in $N(1535)$. Then helicity amplitudes were fairly reproduced, and the ratio $A_{1/2}^n/A_{1/2}^p$ agreed well with experimental data. The success of this calculation without the photon coupling to the possibly quark-originated pole term in the effective interaction implies that the meson-baryon components of $N(1535)$ are essential for the structure of $N(1535)$ proved by low-energy virtual photon.

It is instructive to recall the study of exotic hadrons in the chiral unitary approach [25, 26] where the natural renormalization scheme was adopted. It turned out that the attractive interaction of the WT term in exotic

channels is not strong enough to generate a bound state in the SU(3) limit. As emphasized in the present paper, the natural renormalization scheme, together with the WT term as the interaction kernel, excludes the CDD pole contribution in the scattering amplitude. Thus, the conclusion of Refs. [25, 26] is the absence of the *s*-wave exotic hadrons which are dynamically generated by a meson and baryon without the CDD pole contribution.

VIII. CONCLUSIONS

We have performed a detailed study of the formulation of the chiral unitary approach in order to understand the origin of baryon resonances. We point out that a certain choice for the subtraction constants in the dimensional regularization leads to the positive value of the loop function below threshold. Avoiding this and matching the amplitude with the low-energy interaction, we construct the "natural renormalization" scheme for the loop function in which the CDD pole contribution is excluded. We emphasize again that this scheme is not always applied to the physical scattering system. But rather our aim is to study the structure of the interaction kernel, using the natural renormalization scheme as a starting point.

We then consider the physical meson-baryon scattering with experimental data. We compare the natural renormalization scheme with the phenomenological scheme in which the subtraction constants are fitted to the experimental data keeping the interaction kernel unchanged. From the viewpoint of the renormalization, we show that the same amplitude can be expressed by the natural renormalization scheme with an effective interaction kernel which exhibits a propagator of an elementary particle. This means the necessity of a seed of the resonance in the kernel interaction when the subtraction constant differs from the natural value. This is another mechanism of the CDD pole contribution even if the kernel interaction does not include the contracted resonance propagator in

the low-energy constant. Although both renormalization schemes achieve the same scattering amplitude, the natural scheme is suitable for decomposing the singularity of the amplitude along the same line as the N/D method.

We analyze the $S = -1$ and $S = 0$ meson-baryon scatterings in which the $\Lambda(1405)$ and $N(1535)$ resonances are dynamically generated. Utilizing the phenomenological fitting, we show that $\Lambda(1405)$ can be generated in the natural renormalization scheme with the Weinberg-Tomozawa term, while $N(1535)$ requires substantial correction in addition to the leading order chiral interaction, especially a pole singularity at around 1700 MeV. These facts indicate that $\Lambda(1405)$ can be regarded almost purely as a dynamical state of the meson-baryon scattering, while $N(1535)$ may have an appreciable component originated from quark dynamics, together with the dynamical component as indicated by the coupling properties.

Our analysis can be applied to any system described by the chiral unitary approach. We have also emphasized the importance of the phenomenological fitting to the data, otherwise we cannot extract the correct low-energy structure which is necessary to interpret the origin of the resonance. Hence, precise determination of the meson-hadron scattering data will enable us to further study the properties of hadron resonances.

[1] S. Weinberg, Physica (Amsterdam) **A96**, 327 (1979).
[2] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
[3] A. Pich, Rep. Prog. Phys. **58**, 563 (1995).
[4] G. Ecker, Prog. Part. Nucl. Phys. **35**, 1 (1995).
[5] V. Bernard, N. Kaiser, and U.-G. Meissner, Int. J. Mod. Phys. E **4**, 193 (1995).
[6] S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).
[7] Y. Tomozawa, Nuovo Cimento A **46**, 707 (1966).
[8] N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. **A594**, 325 (1995).
[9] E. Oset and A. Ramos, Nucl. Phys. **A635**, 99 (1998).
[10] B. Krippa, Phys. Rev. C **58**, 1333 (1998).
[11] J. A. Oller and U. G. Meissner, Phys. Lett. **B500**, 263 (2001).
[12] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. **A700**, 193 (2002).
[13] C. Garcia-Recio, M. F. M. Lutz, and J. Nieves, Phys. Lett. **B582**, 49 (2004).
[14] A. Dobado and J. R. Pelaez, Phys. Rev. D **56**, 3057 (1997).
[15] J. A. Oller and E. Oset, Nucl. Phys. **A620**, 438 (1997).
[16] J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. Lett. **80**, 3452 (1998).
[17] J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. D **59**, 074001 (1999).
[18] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. **B585**, 243 (2004).
[19] S. Sarkar, E. Oset, and M. J. Vicente Vacas, Nucl. Phys. **A750**, 294 (2005).
[20] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. **A730**, 392 (2004).

[21] L. Roca, E. Oset, and J. Singh, Phys. Rev. D **72**, 014002 (2005).

[22] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. **B582**, 39 (2004).

[23] F.-K. Guo, P.-N. Shen, H.-C. Chiang, and R.-G. Ping, Phys. Lett. **B641**, 278 (2006).

[24] D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, Phys. Rev. D **76**, 074016 (2007).

[25] T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. Lett. **97**, 192002 (2006).

[26] T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. D **75**, 034002 (2007).

[27] L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 453 (1956).

[28] G. F. Chew and S. C. Frautschi, Phys. Rev. **124**, 264 (1961).

[29] J. A. Oller and E. Oset, Phys. Rev. D **60**, 074023 (1999).

[30] C.-H. Lee, D.-P. Min, and M. Rho, Nucl. Phys. **A602**, 334 (1996).

[31] B. Krippa and J. T. Londergan, Phys. Rev. C **58**, 1634 (1998).

[32] T. Sato and T. S. H. Lee, Phys. Rev. C **54**, 2660 (1996).

[33] K. Igi and K.-i. Hikasa, Phys. Rev. D **59**, 034005 (1999).

[34] D. Jido, E. Oset, and A. Ramos, Phys. Rev. C **66**, 055203 (2002).

[35] G. Ecker, J. Gasser, A. Pich, and E. de Rafael, Nucl. Phys. **B321**, 311 (1989).

[36] J. R. Pelaez, Phys. Rev. Lett. **92**, 102001 (2004).

[37] M. Albaladejo and J. A. Oller, arXiv:0801.4929 [hep-ph].

[38] T. Inoue, E. Oset, and M. J. Vicente Vacas, Phys. Rev. C **65**, 035204 (2002).

[39] B. Borasoy, R. Nissler, and W. Weise, Eur. Phys. J. A **25**, 79 (2005).

[40] J. A. Oller, Eur. Phys. J. A **28**, 63 (2006).

[41] J. A. Oller, J. Prades, and M. Verbeni, Phys. Rev. Lett. **95**, 172502 (2005).

[42] T. Hyodo, S. I. Nam, D. Jido, and A. Hosaka, Phys. Rev. C **68**, 018201 (2003).

[43] T. Hyodo, S. I. Nam, D. Jido, and A. Hosaka, Prog. Theor. Phys. **112**, 73 (2004).

[44] J. Gasser and A. Zepeda, Nucl. Phys. **B174**, 445 (1980).

[45] T. Hyodo, D. Jido, and A. Hosaka, hep-ph/0612333.

[46] T. Hyodo, D. Jido, and A. Hosaka, Prog. Theor. Phys. Suppl. **168**, 32 (2007).

[47] T. Hyodo, D. Jido, and A. Hosaka, arXiv:0710.2853 [hep-ph].

[48] U. G. Meissner and J. A. Oller, Nucl. Phys. **A673**, 311 (2000).

[49] M. Ishida, Prog. Theor. Phys. **96**, 853 (1996).

[50] E. Oset, A. Ramos, and C. Bennhold, Phys. Lett. **B527**, 99 (2002).

[51] D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, Nucl. Phys. **A725**, 181 (2003).

[52] T. Hyodo and W. Weise, Phys. Rev. C, **77**, 035204 (2008).

[53] T. Hyodo, W. Weise, D. Jido, L. Roca, and A. Hosaka, arXiv:0802.2212 [hep-ph], Mod. Phys. Lett. A, (to be published).

[54] S. Weinberg, Phys. Rev. **137**, B672 (1964).

[55] D. Morgan, Nucl. Phys. **A543**, 632 (1992).

[56] V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, and A. E. Kudryavtsev, Phys. Lett. **B586**, 53 (2004).

[57] C. Hanhart, Eur. Phys. J. A **35**, 271 (2008).

[58] T. Hyodo, D. Jido, and L. Roca, Phys. Rev. D, **77**, 056010 (2008).

[59] C. E. DeTar and T. Kunihiro, Phys. Rev. D **39**, 2805 (1989).

[60] D. Jido, Y. Nemoto, M. Oka, and A. Hosaka, Nucl. Phys. **A671**, 471 (2000).

[61] D. Jido, M. Oka, and A. Hosaka, Prog. Theor. Phys. **106**, 873 (2001).

[62] D. Jido, M. Oka, and A. Hosaka, Prog. Theor. Phys. **106**, 823 (2001).

[63] D. Jido, T. Hatsuda, and T. Kunihiro, Phys. Rev. Lett. **84**, 3252 (2000).

[64] D. Jido, M. Doering, and E. Oset, Phys. Rev. C, **77**, 065207 (2008).